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Collective dynamics of a dinuclear system*

E.D. Mshelia[1,a] and W. Scheid[2]

[1] Department of Theoretical Physics, National Mathematical Centre, Abuja, Nigeria
[2] Institut für Theoretische Physik, Justus-Liebig-Universität, Giessen, Germany

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Abstract. There are theoretical and experimental evidences that show that the cluster model, consisting of a dinuclear system, is capable of explaining several features of deformed heavy nuclei. Examples are the fusion to super-heavy nuclei and the nuclear-structure effects arising from parity splitting in alternating rotational parity bands, especially of the actinide nuclei. The model developed by Shneidman *et al.* has been able to explain the parity splitting in the actinides using the cluster model in which oscillations in mass asymmetry and relative motion were considered. On the other hand, Hess and Greiner developed a model for nuclear molecules in which the degrees of freedom of butterfly, belly-dancer–type motions, γ- and β-vibrations of individual nuclei were incorporated. The purpose of this work is to extend the model of Shneidman *et al.* to include some of the degrees of freedom in the work of Hess *et al.* and, in addition, to consider nuclear molecular rotations and the coupling of these degrees of freedom among themselves, arising from mass asymmetry.

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1 Introduction

The cluster model consisting of a dinuclear system has been shown both on theoretical and experimental grounds to be capable of explaining some features of deformed nuclei having cluster properties [1]. Examples of these are the fusion to super-heavy atomic nuclei [2] and nuclear-structure effects such as parity splitting in alternating parity rotational bands of the actinide nuclei [3]. The latter arise from nuclei that possess reflection asymmetric shapes in which the ground-state positive- and negative-parity states considered together do not form undisturbed rotational bands as is the case for asymmetric molecules but in which the negative-parity states are shifted up in energy with respect to the positive-parity states for small spins.

The theory developed by Shneidman, Adamian, Antonenko, Jolos and Scheid, based on the cluster model in which oscillations in mass asymmetry and relative motion were taken into account, has been successful in explaining parity splitting in the actinides [3].

On the other hand, Hess and Greiner developed a collective model of nuclear molecules whereby several degrees of freedom including butterfly, belly-dancer–type motions, γ- and β-vibrations of individual nuclei were included [4]. This model was successfully applied to the ^{238}U-^{238}U system.

The purpose of this work is to extend the model in ref. [3] to include other degrees of freedom described in ref. [4] such as the butterfly motion and the β-vibrations of each dinuclear component as well as rotations of the nuclear molecule and the coupling of these degrees of freedom among themselves, arising from the mass asymmetry of the system.

2 The Hamiltonian

In the description of the formation of nuclear molecules in heavy-ion reactions, deformation effects play an important role. According to the model developed by Greiner and co-workers [5], nuclei approach each other in different orientations and when they touch, the interaction between Coulomb repulsion and nuclear attraction creates a potential bag. The nuclear interaction is simulated by surface interaction. In order for contact to be possible, different energies are required for distinct orientations. In extreme cases such as the pole-pole and equator-equator orientations of the nuclei, the Coulomb energy during contact is least and largest, respectively, due to different internuclear distances. Because the overlap in surface area is

* Work supported by the German Academic Exchange Service (DAAD), Bonn, Germany.

[a] *Permanent address*: Department of Physics, Abubakar Tafawa Balewa University, Bauchi, Nigeria; e-mail: `emshelia@atbu.edu.ng`

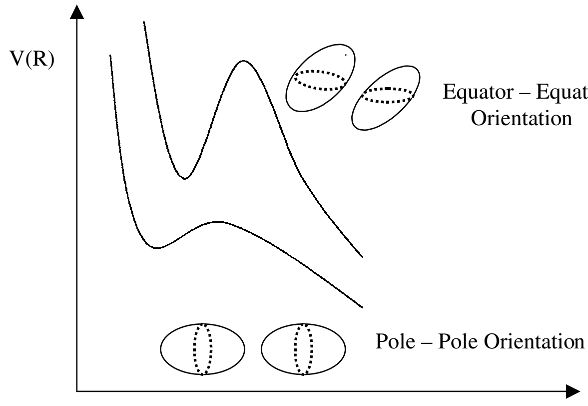

Fig. 1. Schematic behaviour of the dinuclear potentials as a function of the relative distance R and dinuclear orientation. Because the overlap in surface area is greater in the equator-equator orientation, the nuclear attraction is stronger and, consequently, the molecular potential has a deeper well than in the pole-pole orientation.

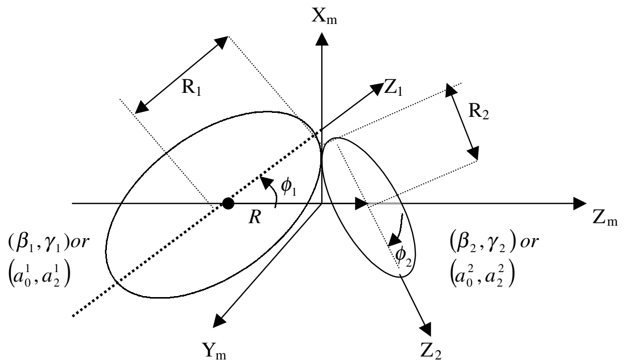

Fig. 2. Illustration of the molecular reference frame at the touching point, showing the symmetry and principal axes of rotation. The angles ϕ_1 and ϕ_2 designate the orientations of the nuclei to the molecular axis Z_{m} along which the relative separation of centres $\boldsymbol{R}$ is defined.

greater in the equator-equator orientation, the nuclear attraction is stronger and, consequently, the molecular potential is deeper than in the pole-pole orientation (see fig. 1). In this work we shall consider the pole-pole orientation, as it is lower in energy than the equator-equator orientation.

In order to define the butterfly degree of freedom, we refer to fig. 2 [4]. The molecular frame is represented by the coordinates X_{m}, Y_{m}, Z_{m}. The system of principal axes of the nuclei in general does not coincide with the molecular frame. The distance $\boldsymbol{R}$ between the nuclei is defined along the molecular Z_{m}-axis and it is assumed that $(R-R_0) \ll R_0$, where R_0 is the minimum position of the relative potential energy as a function of R.

For prolate nuclei, as being considered here, R_1 and R_2 denote the radii along their respective symmetry axes. Let ϕ_1 and ϕ_2 be the orientations of the nuclei with respect to the molecular frame. Assuming that the angles are small, *i.e.* $|\phi_1| \ll 1$ and $|\phi_2| \ll 1$, there results

$$\frac{\sin|\phi_2|}{\sin|\phi_1|} \approx \frac{R_1}{R_2} \qquad \text{or} \qquad |\phi_2| \approx |\phi_1|\frac{R_1}{R_2}\,.$$

But ϕ_2 has the opposite sign of ϕ_1, and choosing $\phi_1 = \varepsilon$ to be the butterfly mode, we obtain $\phi_2 = -(R_1/R_2)\,\varepsilon$. The notations (β_i, γ_i) or (a_0^i, a_2^i), $i = 1, 2$, denote β- and γ-vibrations, respectively, of each dinuclear component.

Other approximations involve expanding the moments of inertia of each dinuclear component in terms of $B_i\beta_{0i}^2/(\mu R_0^2)$, where β_{0i} $(i = 1, 2)$ are the ground-state deformation parameters, B_i $(i = 1, 2)$ the collective inertia parameters and μ the reduced mass [4]. The $B_i\beta_{0i}^2/(\mu R_0^2)$ quantity is of the order of 1/30 for all nuclei and hence the expansion of the moments of inertia is a good approximation [4].

As earlier indicated, the dinuclear system is considered to be asymmetric, with each nucleus having different radii R_1 and R_2. In addition, apart from the butterfly motion, we consider here only β-vibrations without γ-vibrations.

The total Hamiltonian operator has the following structure:

$$\hat{H} = \hat{H}_{\eta R} + \hat{H}_{\eta\varepsilon\mathrm{rot}} + \hat{H}_{\xi 1} + \hat{H}_{\xi 2}\,, \tag{1}$$

where

$$\hat{H}_{\eta R} = -\frac{\hbar^2}{2B_{RR}}\frac{\partial^2}{\partial R^2} - \frac{\hbar^2}{2B_{\eta\eta}}\frac{\partial^2}{\partial \eta^2} - \frac{\hbar^2}{B_{\eta R}}\frac{\partial^2}{\partial R\partial\eta} + U(R,\eta) \tag{2}$$

describes the coupling between the mass asymmetry degrees of freedom η, defined by $\eta = (A_1 - A_2)/(A_1 + A_2)$, and the relative motion described by R. The potential $U(R, \eta)$ describes the potential energy of the dinuclear system.

The operator

$$\begin{aligned}\hat{H}_{\eta\varepsilon\mathrm{rot}} = {} & \frac{\hbar^2}{2\mu R^2}(\hat{L}^2 - \hat{L}_3'^2) + \frac{\hbar^2 \hat{L}_3'^2}{6\left[B_1\beta_{01}^2 + \left(\frac{R_1}{R_2}\right)^2 B_2\beta_{02}^2\right]\varepsilon^2} \\ & - \frac{\hbar^2}{6\left[B_1\beta_{01}^2 + \left(\frac{R_1}{R_2}\right)^2 B_2\beta_{02}^2\right]}\frac{\partial^2}{\partial\varepsilon^2} \\ & - \frac{\hbar^2}{24\left[B_1\beta_{01}^2 + \left(\frac{R_1}{R_2}\right)^2 B_2\beta_{02}^2\right]\varepsilon^2} + \frac{1}{2}C_\varepsilon\varepsilon^2 \end{aligned} \tag{3}$$

describes rotations of the molecular system and the coupling of rotations to the butterfly motion as well as to mass asymmetry, implicitly contained in the reduced mass μ, the collective inertia parameter B_i $(i = 1, 2)$ and the dinuclear radii R_i $(i = 1, 2)$ (see later). The quantity C_ε is the stiffness parameter for the butterfly motion. Other quantities appearing in eq. (3) include the total angular momentum $\hat{\boldsymbol{L}}$ and the third component of the angular momentum $\hat{L}_3'$ along the rotating intrinsic (body-fixed) symmetry axis joining the centres of the two nuclei.

The Hamiltonian operators occurring in eq. (1),

$$\hat{H}_{\xi_i} = -\frac{\hbar^2}{2B_i}\frac{\partial^2}{\partial\xi_i^2} + \frac{1}{2}C_{\xi_i}\xi_i^2 \qquad (i = 1, 2) \tag{4}$$

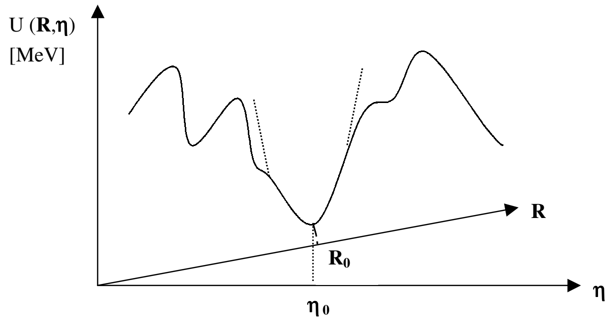

Fig. 3. Schematic representation of the dinuclear potential energy as a function of R and η, showing the minimum at R_0 and η_0 about which $U(R,\eta)$ may be expanded.

describe the β-vibrations in the harmonic approximation of each dinuclear component.

The eigenvalue equation arising from the application of the total Hamiltonian operator (1) on an eigenstate may be solved by the method of the time-independent perturbation theory. For this to be possible a few more approximations need to be introduced, so that the total Hamiltonian can be reduced to an unperturbed part that is simple and straightforward to solve exactly, and a small part that can be solved by the method of stationary perturbations.

First, we neglect in eq. (2) the term proportional to the inverse mass $B_{\eta R}$, since this has been shown to be negligible compared to the others [6].

Second, we expand the potential energy $U(R,\eta)$ in eq. (2) in powers of $R-R_0$ and $\eta-\eta_0$ about the minimum value of the potential at $R=R_0$ and $\eta=\eta_0$ up to second-order terms (see fig. 3).

Third, the masses μ, B_{RR} and $B_{\eta\eta}$ depend on the mass asymmetry coordinate η and are defined by [6]

$$\left.\begin{aligned} \mu &= \tfrac{M}{4}(1-\eta^2), \\ B_{RR} &= \tfrac{M}{4}(1-\eta^2), \\ B_{\eta\eta} &= \tfrac{M}{4}\left(\tfrac{V}{\pi R_{\mathrm{n}}^2}R - R^2\right), \end{aligned}\right\} \tag{5}$$

where M is the total mass, V is the total volume and R_{n} is the neck radius during mass transfer in heavy-ion collisions. We assume small orientations of the nuclei with respect to the molecular frame. The radii of the nuclei are given by

$$R_i = r_{0i}A_i^{1/3} = r_{0i}\left(\frac{A}{2}\right)^{1/3}(1\pm\eta)^{1/3}, \quad i=1,2. \tag{6}$$

Since the masses appear as inverses in the expression for the Hamiltonian, an expansion of $(1-\eta^2)^{-1}$ about $\eta=\eta_0$ is made up to first order of small quantities.

Finally, because the nuclear radii are defined by eq. (6), an expansion of (R_1/R_2) which appears in the Hamiltonian (eq. (3)) may be made about $\eta=\eta_0$ up to first-order terms in $(\eta-\eta_0)$.

When these approximations are taken into account, the total Hamiltonian operator defined in eq. (1) splits into two parts: a main part (unperturbed Hamiltonian) and a small part (perturbed Hamiltonian). This may be cast in the form

$$\hat{H} = \hat{H}^{(0)} + \hat{H}', \tag{7}$$

where

$$\hat{H}^{(0)} = \hat{H}_R^{(0)} + \hat{H}_\eta^{(0)} + \hat{H}_{\eta\varepsilon\mathrm{rot}}^{(0)} + \hat{H}_{\xi_1} + \hat{H}_{\xi_2}, \tag{8}$$

$$\hat{H}_R^{(0)} = -\frac{\hbar^2}{2\mu_0}\frac{\partial^2}{\partial R^2} + \frac{1}{2}C_{R_0}(R-R_0)^2, \tag{9}$$

$$\hat{H}_\eta^{(0)} = -\frac{\hbar^2}{2p}\frac{\partial^2}{\partial\eta^2} + \frac{1}{2}C_{\eta_0}(\eta-\eta_0)^2 + U(R_0,\eta_0), \tag{10}$$

$$\begin{aligned} \hat{H}_{\eta\varepsilon\mathrm{rot}}^{(0)} = {} & \frac{\hbar^2}{2\mu_0 R_0^2}(\hat{L}^2 - \hat{L}_3'^2) \\ & + \frac{\hbar^2(4\hat{L}_3'^2-1)}{24\left[B_1\beta_{01}^2 + \left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{02}^2\right]\varepsilon^2} \\ & - \frac{\hbar^2}{6\left[B_1\beta_{01}^2 + \left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{02}^2\right]}\frac{\partial^2}{\partial\varepsilon^2} + \frac{1}{2}C_\varepsilon\varepsilon^2. \end{aligned} \tag{11}$$

The constant $p = B_{\eta\eta}(R_0,\eta_0)$ occurring in eq. (10) is defined in terms of eq. (5), while $\mu_0 = \frac{M}{4}(1-\eta_0^2)$. The Hamiltonian H_{ξ_i} $(i=1,2)$ has been defined in eq. (4).

The perturbed Hamiltonian is given by

$$\hat{H}' = \hat{H}'_R + \hat{H}'_\eta + \hat{H}'_{\eta\varepsilon\mathrm{rot}}, \tag{12}$$

where

$$\hat{H}'_R = -\frac{\eta_0(\eta-\eta_0)}{1-\eta_0^2}\frac{\hbar^2}{\mu_0}\frac{\partial^2}{\partial R^2}, \tag{13}$$

$$\hat{H}'_\eta = \frac{\hbar^2}{2p^2}(q_1(\eta-\eta_0) + q_2(R-R_0))\frac{\partial^2}{\partial\eta^2}, \tag{14}$$

where we introduced $q_1 = (\partial B_{\eta\eta}/\partial\eta)_{R_0,\eta_0}$ and $q_2 = (\partial B_{\eta\eta}/\partial R)_{R_0,\eta_0}$ with $B_{\eta\eta}(R,\eta)$ defined in eq. (5).

Furthermore, we have

$$\begin{aligned} \hat{H}'_{\eta\varepsilon\mathrm{rot}} = {} & \left(\frac{\eta_0(\eta-\eta_0)}{1-\eta_0^2} - \frac{R-R_0}{R_0}\right)\frac{\hbar^2}{\mu_0 R_0^2}(\hat{L}^2 - \hat{L}_3'^2) \\ & + Q\left(\frac{\eta-\eta_0}{1-\eta_0^2}\right)\left\{\frac{1}{6}\frac{\partial^2}{\partial\varepsilon^2} - \frac{(4\hat{L}_3'^2-1)}{24\varepsilon^2}\right\}, \end{aligned} \tag{15}$$

where the constant Q is defined by

$$Q = \frac{\frac{4}{3}\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{02}^2\hbar^2}{\left[B_1\beta_{01} + \left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{01}^2\right]^2}. \tag{16}$$

3 Solution of the eigenvalue problem

The eigenvalue equation of the unperturbed Hamiltonian operator is

$$\hat{H}^{(0)}\big|\varphi_n^{(0)}\big\rangle = E_n^{(0)}\big|\varphi_n^{(0)}\big\rangle\,. \quad (17)$$

Similarly to the procedure outlined in ref. [4], we assume a separable product solution of the form

$$\varphi_n^{(0)}(\theta_j,\eta,\varepsilon,R,\xi_1,\xi_2) = D_{MK}^{I^*}(\theta_j)\chi(\varepsilon)g(\eta)g(R)g(\xi_1)g(\xi_2), \quad (18)$$

where θ_j $(j=1,2,3)$ denote the Eulerian angles.

The g-functions occurring in eq. (18) are the linear harmonic-oscillator wave functions because their corresponding Hamiltonians $\hat{H}_R^{(0)}, \hat{H}_\eta^{(0)}, \hat{H}_{\xi_1}, H_{\xi_2}$ defined, respectively, by eqs. (9), (10) and (4) are harmonic-oscillator Hamiltonians.

In view of the eigenvalue equation,

$$\left\{\frac{\hbar^2}{2\mu_0 R_0^2}(\hat{L}^2-\hat{L}_3'^2) + \frac{\hbar^2\hat{L}_3'^2}{6\left[B_1\beta_{01}^2+\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{01}^2\right]\varepsilon^2}\right\}D_{MK}^{I*}(\theta_j) =$$

$$\left\{\frac{\hbar^2}{2\mu_0 R_0^2}\left[I(I+1)-K^2\right] + \frac{\hbar^2 K^2}{6\left[B_1\beta_{01}^2+\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{01}^2\right]\varepsilon^2}\right\}D_{MK}^{I*}(\theta_j)\,, \quad (19)$$

equation (17) reduces to ($\ell_K = |K|-0.5$)

$$\left\{\frac{\partial^2}{\partial\varepsilon^2} - \wedge^2\varepsilon^2 - \frac{\ell_K(\ell_K+1)}{\varepsilon^2} + \kappa^2\right\}\chi(\varepsilon) = 0\,, \quad (20)$$

where

$$\wedge^2 = \frac{3}{\hbar^2}\left[B_1\beta_{01}^2+\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{02}^2\right]C_\varepsilon\,, \quad (21)$$

$$\kappa^2 = \frac{6}{\hbar^2}\left[B_1\beta_{01}^2+\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{02}^2\right] \times\left[E_n^{(0)} - \lambda_0 - \frac{\hbar^2}{2\mu_0 R_0^2}\left\{I(I+1)-K^2\right\}\right], \quad (22)$$

$$\left.\begin{aligned} \lambda_0 &= E_{n_R} + E_{n_\eta} + E_{n_{\xi_1}} + E_{n_{\xi_2}} + U(R_0,\eta_0)\,,\\ E_{n_R} &= \hbar\omega_{n_R}\left(n_R+\tfrac{1}{2}\right),\\ E_{n_{\xi_i}} &= \hbar\omega_{n_i}\left(n_{\xi_i}+\tfrac{1}{2}\right), \quad i=1,2\,,\\ E_{n_\eta} &= \hbar\omega_{n_\eta}\left(n_\eta+\tfrac{1}{2}\right). \end{aligned}\right\} \quad (23)$$

Equation (20) describes the butterfly motion of the dinuclear system. It is the differential equation of the spherical oscillator. Its structure is similar to that of the well-known rotation-vibration model of deformed even-even nuclei which has been extensively discussed by Eisenberg and Greiner [7]. The solutions are given by

$$\chi_{K,n_\varepsilon}(\varepsilon) = C_1\varepsilon^{\ell_K+1}\exp(-\wedge\varepsilon^2/2) \times {}_1F_1\left[\frac{1}{2}\left(\ell_K+\frac{3}{2}-\frac{\kappa^2}{2\wedge}\right);\ell_K+\frac{3}{2};\wedge\varepsilon^2\right], \quad (24)$$

where C_1 is a normalization constant and the quantity ${}_1F_1$ is a confluent hypergeometric function,

$$\ell_K = \frac{1}{2}(2|K|-1)\,, \quad (25)$$

$$E_n^{(0)} = \lambda_0 + \left(\ell_K+\frac{3}{2}+2n_\varepsilon\right)\hbar\omega_{n_\varepsilon} + \left[I(I+1)-K^2\right]\frac{\hbar^2}{2\mu_0 R_0^2}\,. \quad (26)$$

Furthermore, we have

$$\hbar\omega_{n_\varepsilon} = \hbar\left[\frac{C_\varepsilon}{3}\Big/\left\{B_1\beta_{01}^2+\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{02}^2\right\}\right], \quad (27)$$

where $n_\varepsilon = 0,1,2,\ldots$.

From eq. (26) and the definition of λ_0 in eq. (23) we see that there exist β-vibrations in the ξ_1 and ξ_2 degrees of freedom that are analogous to those of normally deformed nuclei in addition to R- and η-vibrations, while the motion in the butterfly degree of freedom, is analogous to γ-vibrations of normally deformed nuclei. Superposed on these vibrations are the rotations of the dinuclear system about the Y_{m}-axis.

The quantum numbers I and K in eq. (26) originate from the rotational term $\hbar^2(\hat{L}^2-\hat{L}_3'^2)/2\mu_0R_0^2$ and the coupling term $\hbar^2\hat{L}_3'^2\Big/\left(6\left[B_1\beta_{01}^2+\left(\frac{1+\eta_0}{1-\eta_0}\right)^{2/3}B_2\beta_{01}^2\right]\varepsilon^2\right)$ between rotations and butterfly motion. The angular momentum $\hat{L}_3'$ refers to the molecular symmetry axis.

The symmetrized wave function of the system has the form

$$\begin{aligned}\big|\varphi_n^{(0)}\big\rangle &= \big|I\,M\,K\,n_\eta n_R n_{\xi_1} n_{\xi_2} n_\varepsilon\big\rangle = \\ &C\left[D_{MK}^{I*}(\theta_j) + (-1)^{I-K}D_{M-K}^{I*}(\theta_j)\right] \\ &\times\chi_{K,n_\varepsilon}(\varepsilon)g_{n_\eta}(\eta)g_{n_R}(R)g_{\xi_1}(\xi_1)g_{\xi_2}(\xi_2)\,, \end{aligned} \quad (28)$$

where C is a normalization constant, defined by

$$C = \sqrt{\frac{(2I+1)}{16\pi^2(1+\delta_{KO})}}\,. \quad (29)$$

The quantum numbers K and I are restricted by the following values:

$$\left.\begin{aligned} K &= 0,1,2,3,\ldots\\ I &= 0,2,4,6,\ldots, \quad \text{if } K=0\\ I &= K,\,K+1,K+2,\ldots, \quad \text{if } K\neq 0 \end{aligned}\right\}. \quad (30)$$

Thus, band heads may be classified by the 6 quantum numbers: K, n_η, n_R, n_{ξ_1}, n_{ξ_2}, n_ε.

Since the solution of the unperturbed Hamiltonian, $\hat{H}^{(0)}$ defined in eqs. (7) and (8) has been analytically determined, the solution of the total Hamiltonian, $\hat{H}$, may be found by the usual methods of the time-independent perturbation theory which may be represented as follows:

$$\left|\Psi_n\right\rangle = \left|\varphi_n^{(0)}\right\rangle + \sum_{m\neq n} \frac{\left\langle \varphi_{\mathrm{m}}^{(0)}\middle|\hat{H}'\middle|\varphi_n^{(0)}\right\rangle}{E_n^{(0)} - E_{\mathrm{m}}^{(0)}}\left|\varphi_{\mathrm{m}}^{(0)}\right\rangle, \tag{31}$$

$$E_n = E_n^{(0)} + \left\langle \varphi_n^{(0)}\middle|\hat{H}'\middle|\varphi_n^{(0)}\right\rangle + \sum_{m\neq n} \frac{\left|\left\langle \varphi_n^{(0)}\middle|\hat{H}'\middle|\varphi_{\mathrm{m}}^{(0)}\right\rangle\right|^2}{E_n^{(0)} - E_{\mathrm{m}}^{(0)}}, \tag{32}$$

provided that

$$\left|\frac{\left\langle \varphi_n^{(0)}\middle|\hat{H}'\middle|\varphi_{\mathrm{m}}^{(0)}\right\rangle}{E_n^{(0)} - E_{\mathrm{m}}^{(0)}}\right| \ll 1\,. \tag{33}$$

In eqs. (31) and (32), the wave function is given to first order while the energy is determined to second order. For $\hat{H}'$ the first-order perturbation of energy vanishes.

4 Summary and conclusion

The cluster model of a dinuclear system has been successful in explaining several features of nuclear phenomena including fusion to super-heavy nuclei and parity splitting in alternate parity bands of the actinides. The work of Shneidman, Adamian, Antonenko, Jolos and Scheid gives testimony to these facts.

In this work we have extended the model proposed by Shneidman *et al.* to take into account other collective modes of nuclear molecules proposed by Hess and Greiner. Apart from the relative motion of the dinuclear system and mass asymmetry degree of freedom, we introduced the butterfly motion, the β-vibrations of each nucleus as well as rotational motion and considered the case when the dinuclear system may be asymmetric. The nucleus-nucleus potential, the masses and the dinuclear radii were expanded in power series about $R = R_0$ and $\eta = \eta_0$ as an approximation. This enabled us to split the total Hamiltonian into an unperturbed part that can be solved analytically by simple means and a perturbed part that may be solved by perturbation techniques. The solution of the unperturbed Hamiltonian consists of linear harmonic vibrations in relative coordinate R, mass asymmetry η and β-degrees (ξ_1, ξ_2) of freedom, while the butterfly motion, represented by the coordinate ε, is described by a spherical oscillator. Superimposed on these vibrations are the rotations of the dinuclear system about an axis perpendicular to the molecular asymmetry axis.

Our model no doubt represents an improvement on both of the models of Shneidman *et al.* and Hess *et al.*, since the aspects of each of the latter models have been taken into account in our model. We therefore expect our model to be able to explain not only nuclear-structure effects such as the parity splitting in alternating parity bands of the actinides, but also that with the wave functions that may be calculated, one should be able to determine other quantities of interest such as transition probabilities from one dinuclear level to the other. Work along this line is in progress. Finally, an inclusion of γ-vibrations and "belly-dancer" modes in this model would be straightforward.

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